Teaching and Learning Statement

Paul Hartzer

Position statement: *An effective mathematics classroom focuses on the goal of each student's complete subject mastery by being intellectually engaging, cognitively accessible, and emotionally supportive.*

Theoretical Background

"Give a man a fish and you feed him for a day," says a famous proverb. "Teach a man to fish and you feed him for a lifetime." Mathematics education consists of two distinct components: Teaching facts (such as $2+2=4$ and $d(\sin x) = \cos x\,dx$) and guiding ways of thinking and solving problems. Facts are important, but inasmuch as mathematics is a predominantly theoretical field, a way of arranging phenomena about the world into a meaningful network of patterns, its facts ultimately come from its theories. Those who know the facts but do not have the strategies have difficult producing new facts; those who have sound strategies have a way of developing an infinity of new facts.

I believe the primary role of the mathematics instructor to act as a guide by encouraging students to discover and develop new strategies for approaching mathematical problems. This crucially relies on leveraging the students' existing knowledge base as a way of building these new strategies. A secondary role of the instructor is to present enough facts, in a constructive way, to help students see additional patterns and strategies.

For example, consider the following table, typical of what an advanced high school student or college freshman might see in a calculus course:

$f(\theta)$	$f'(\theta)$		$f(\theta)$	$f'(\theta)$
$\sin\theta$	$\cos\theta\,d\theta$		$\cot\theta$	$-\csc^2\theta\,d\theta$
$\cos\theta$	$-\sin\theta\,d\theta$		$\sec\theta$	$\sec\theta\tan\theta\,d\theta$
$\tan\theta$	$\sec^2\theta\,d\theta$		$\csc\theta$	$-\csc\theta\cot\theta\,d\theta$

There are two basic approaches to learning such a table. A student can be expected to simply memorize the entire table and bring it up on demand, or the student can learn basic facts about trigonometry and calculus sufficient to generate the table as needed but also useful in other applications, such as these:

- Prior knowledge (trigonometry)
  - $\tan\theta = {}^{\sin\theta}/_{\cos\theta}$
  - $\cot\theta = {}^{1}/_{\tan\theta} = {}^{\cos\theta}/_{\sin\theta}$
  - $\sec\theta = {}^{1}/_{\cos\theta}$
  - $\csc\theta = {}^{1}/_{\sin\theta}$
  - $\sin^2\theta + \cos^2\theta = 1$
- Quotient Rule (basic calculus)
  - $d(f(x)/g(x)) = \dfrac{g(x)f'(x) - g'(x)f(x)}{g^2(x)}$
- Specific knowledge (calculus)
  - $d(\sin\theta) = \cos\theta\, d\theta$
  - $d(\cos\theta) = -\sin\theta\, d\theta$

The identity $\sin^2\theta + \cos^2\theta = 1$ can in turn be derived from the Pythagorean theorem ($a^2 + b^2 = c^2$) and the definition of $\sin\theta$ and $\cos\theta$ as ${}^{a}/_{c}$ and ${}^{b}/_{c}$, respectively. Furthermore, the Quotient Rule can be derived from the Power Rule ($d(f(x^n)) = nx^{n-1}\,dx$) and the Product Rule ($d(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$). Simply teaching each of these things as rules to be followed, without at least offering the underlying derivation, obstructs full mastery on the part of the student.

In my honors calculus course, I have used the following example to demonstrate that the integral really does calculate the area under a curve. Take the function $y = 2x$, from 0 to 4:

$$y = \int_0^4 2x\,dx$$

This is easy to integrate by the inverse of the Power Rule, specifically:

$$\begin{aligned} y &= \int_0^4 2x\,dx \\ &= \left.\frac{2x^2}{2}\right|_0^4 = \left.x^2\right|_0^4 \\ &= 4^2 - 0^2 = 16 \end{aligned}$$

Consider that $y = 2x$ represents the hypotenuse of a triangle which passes through (0,0) and (4,8), in other words, a right triangle with a base of 4 and a height of 8. From geometry, students know how to calculate this area: $h \cdot w/2 = 8 \cdot 4/2 = 32/2 = 16$. They can thus see that this new thing, integration, returns the same result as a known method, and they feel more confident trusting integration for more complex functions.

Mathematics education is particularly notorious for being boring, difficult to penetrate, and little but a series of meaningless algorithms and rules. By showing how these algorithms and rules were developed in the first place, the effective educator can guide students towards mastery, as well as kindling the flame of curiosity for further exploration: Ultimately, nearly every rule of mathematics was devised by sound, systematic problem-solving. One strong challenge that the secondary mathematics educator must overcome is the experience and self-efficacy of many students that they can't do math, that they're just not any good at it.

One way to improve self-efficacy is through group-work, as struggling students see both that some of their cohort also struggle and that some of their cohort have mastered parts of the material. A study by Bishop and Pflaum showed that students have more positive attitudes towards student-focused activity work than lecture-based lessons.[1] Additionally, direct group work leads to higher retention; while lecture-based education can have retention as low as 5%, long-term retention of hands-on group work is about 50%.[2]

My ideal classroom management and organization is constructed to facilitate student exploration. Desks should be easy to move between a lecture-focussed grid style (preferably with semi-circular rows rather than straight ones, allowing for greater student interaction) and group-based desk clustering. Rules are based on mutual respect between students, to encourage all students to participate. The message through both the desk layout and the rules is that, while the designated teacher may have more content-specific knowledge, everyone in the room is there to contribute, to teach, and to learn.

A significant portion of my assessment of student understanding will be based on close monitoring of participation, enthusiasm, and consistency of student results. While a certain portion of the grade must be based on traditional summative exercises, strong attention will also be paid to formative exercises. Examples include completion of worksheets and bellwork, "show of hands" and accuracy of solutions completed on the board, degree and quality of participation in group work, and so forth.

[1] Willis, J. (2010) *Learning to Love Math: Teaching Strategies that Change Student Attitudes and Get Results*. Alexandria, VA: ASCD. p. 10.

[2] Meriweather, A. (2011, July). Class notes, TED6350 (Effective Urban Educator).

Reflectiveness

Learning is a communal activity, involving dialogue between the teacher and the student. A teacher presents material; an effective teacher ensures that the material is being understood by the students, and this can only be accomplished by truly listening to the students as they articulate their understanding of that material.

My personal background earning a Master's degree in linguistics brings with it a sensitivity to the transition points in a message: The teacher speaks, the student interprets; the student responds, the teacher interprets. A simple exchange thus represents four transition points, four places in which communication can be effected or disrupted. As the professional in the situation, it is incumbent particularly on the teacher to be mindful of each of these transition points. It is not nearly enough to merely present information and assume that the "smart" students will get it and the students who don't get it just aren't trying.

Overcoming this assumption requires a high degree of self-awareness on the teacher's part. Formative assessments serve a dual purpose: They allow the teacher to gauge student mastery, but on a deeper level, they allow the teacher to explore the rift between teaching goals and actual accomplishments. The consistent failure of students to master material as intended is ultimately rooted in the teacher's lapse in presenting the material in an effective way.

As a teacher, I feel that I effectively monitor body language and other cues to gauge levels of understanding, and that I'm flexible with modifying content to match student needs. A weakness at this early stage of my career is that I can sometimes rush through material in an attempt to "get through it all"; as I continue to develop as a teacher, I will place a premium not on getting through all the planned material but rather on focusing on student understanding.

Another personal weakness is due to the fact that I have always excelled at mathematics, meaning that it's sometimes difficult for me to relate fully to those students who have more challenges in that regard. I do believe that every student has the capacity to master mathematics, particularly at the high school level. However, I can sometimes rush through explanations while struggling students just fall farther behind. To address this, I have used tutorial opportunities to explore how students learn, and where the disconnect is between them and the material, in order to better manage how I teach at the class level.

As a part of my educational experience, I attended DACTM '11[3]. As I discuss in the next session, I used the material presented in one of the lectures there in my pre-calculus class. Additionally, I have visited a variety of schools around the Detroit Public School system, as well as sitting in on several classes in subjects other than mathematics at my current pre-service placement, Western

[3] Detroit Area Council of Teachers of Mathematics and Metropolitan Detroit Science Teachers Association 2011 Fall Conference

International High School. This experience have given me a variety of perspectives on how to teach (and not teach).

Innovation

As discussed above, simply lecturing on a subject is the least effective way to teach true mastery. Educational experiences must be dynamic and involve student participation as both individuals and members of a learning community. Different students learn differently, and it's incumbent upon the educational facilitator to reach out across the multiple intelligences.

One way to be innovative is to tie math topics in to real world examples. When early mathematicians developed ways of calculating the circumference of circles, it wasn't to pass the time or muse on universals, it was to provide practical ways to, for instance, use wheels to measure distance. In his Commentary on Euclid's Elements, for instance, Proclus describes how geometry came from the Egyptian need to calculate the impact of the Nile flooding.[4]

Students interact with math every day. Athletes deal with physics in calculating, for instance, whether they can make it to second base before the ball does. Students buy things: How do they calculate whether they have enough money for what they want? Students' walks to school can be used to teach trigonometry; students' clothing sizes or personal preferences can be used to teach statistics, and to discuss market research.

Another way in which innovation is used in my classroom is through reinforcing that the most important aspect of mathematics is getting the same answer from the same problem, regardless of the choice of intervening steps. For instance, as discussed above, either trigonometry or calculus can be used to find the area of the polygon. Elementary and secondary students might be aware of a variety of ways to do long multiplication, such as:

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  45
  32
————
  90
135x
————
1440
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  45
  32
————
  10
  8x
 15x
12xx
————
1440
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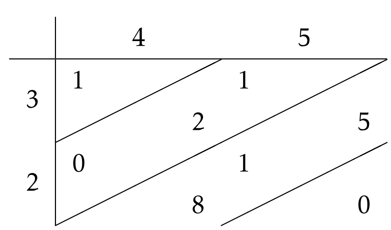

1000+300+140+0=1440

	40	5
30	1200	150
2	80	10

1200+150+80+10
= 1440

[4]Frankland, W. B. (1905) *The first book of Euclid's elements: with a commentary based principally upon that of Proclus Diadochus*. London: Cambridge University Press Warehouse. p. ix.

All of these methods return the same result, even though they look different. The last of these involves a table that can be generalized for polynomial multiplication and division, as well as several other related functions.[5] I presented this alternative to traditional polynomial multiplication and division to my pre-calculus students; for instance, $(2x^2 - 5x + 7)(4x^2 + 3x + 1)$ would be represented and solved as follows:

	$2x^2$	$-5x$	$+7$
$4x^2$	$8x^4$	$-20x^3$	$+28x^2$
$+3x$	$+6x^3$	$-15x^2$	$+21x$
$+1$	$+2x^2$	$-5x$	$+7$

generating the final answer of $(2x^2 - 5x + 7)(4x^2 + 3x + 1) = 8x^4 - 14x^3 + 15x^2 + 16x + 7$. By presenting students with multiple solving strategies, and explicitly directing them to use the one that works best for them, I reinforce the core theme that there's rarely a single "correct" way to solve a problem.

Commitment to Diversity

Different students learn in different ways. Some students come into the classroom expecting to excel in mathematics, others come equipped thinking that they're no good at it. Each student brings in different life experiences, and different challenges from home can bleed over into the learning environment. Some students bring language barriers, learning barriers, or physical or mental impairments that create obstacles for content mastery.

One way in which I can address some of these challenges is by gathering information about each of my students. What is their neighborhood like? What are their cultural expectations for success? What sort of formal IEP-based restrictions do they have? It is important for me to understand as much as possible about where each student is coming from, and to make accommodations where I can. Additionally, I encourage parents and other caregivers to be involved in the educational process, either directly through helping their charges in the work or indirectly through providing safe, quiet places to complete homework.

Within the context of the classroom, as much as possible, all students will be treated equally while being respected as individuals with different strengths and weaknesses. If the goal is for everyone to reach the same place (subject mastery), and everyone starts from a different place, then it stands to reason that they'll have to take different paths to get to the goal. My objective is to find those paths and aid the students along them.

[5] Imboden, D. (2011) "FOIL Is Dead! Use Generic Rectangles to Unify Eight Algebra 1 and 2 Skills". Presented at Detroit Area Council of Teachers of Mathematics and Metropolitan Detroit Science Teachers Association 2011 Fall Conference, November 12, 2011.

Accommodations can include subtle things, such as the use of color in classroom presentations to highlight important differences and the careful use of words to minimize confusion, particularly for students who have not fully mastered English. While I place an emphasis on group work, it must be carefully constructed to be useful: Each group should include students who are both strong and weak on the current subject matter, for instance. Also, I generally avoid placing students with known interpersonal conflicts in the same group; while I understand that we can't choose our co-workers in the corporate world, that is trumped by the need for safety in the classroom.

Furthermore, a good educator is sensitive to the cultural background of their students, especially when it contrasts with the material in their textbooks. Texts and standardized test questions have been criticized for being focused around the life experience of middle-class Americans with European backgrounds, and even when there are attempts to address this, those attempts can backfire. For instance, in the geometry text used in Detroit Public Schools, there is an example involving a carpenter named Juan. My Latino students responded to this with despair, making "jokes" about how this was what they were destined to do, so they should pay attention. In developing my own "real world" examples, I am cautious to provide a set of references that is either culturally unbiased or specifically appeals to my students' backgrounds and life experiences in a positive way.

When it comes to assessment, every student has different needs and learning patterns. Some students perform well on summative exercises but struggle in participation; others are the opposite. I believe the fairest route is a middle ground: A significant portion of the student's assessment is based on traditional concepts such as tests, while another portion of the student's assessment is based on their demonstration of the concepts in more dynamic settings. It would be difficult for me to fail a student who has demonstrated through classroom participation or one-on-one discussion that they understand the material, but who fails when faced with traditional tests that are atypical of real world applications in the first place. When faced with such a student, I might offer alternative or extra credit assignments allowing them to demonstrate their understanding in a tangible, documented way.